they all went anew, for what it contained; of which, A got $\frac{1}{4}$, B $\frac{1}{3}$, and D $\frac{2}{7}$, and C and E equal shares of what was left of that stock. D then struck $\frac{3}{4}$ of what A and B last acquired, out of their hands; they, with difficulty, recovered $\frac{5}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{8}$ apiece of the same. Upon this, they called a truce, and agreed that the $\frac{1}{3}$ of the whole, left by A at first, should be equally divided among them. How much of the prize, after this distribution, remained with each of the competitors?

set upon B, who, in the conflict, let fall $\frac{1}{2}$ he had, which were equally picked up by D and E, who lay perdu. B then kicked down C's hat, and to work

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

32. Proposed by LEV. WEINER, Professor of Modern Languages, Missouri State University, Columbia, Missouri.

Find a number consisting of 6 digits which when multiplied by the first 6 natural numbers gives the same digits in rotation.

I. Solution by LEONARD E DICKSON, M. A., Fellow in Mathematics, University of Chicago.

In a memoir on "Numbers with cyclic multiples" soon to be published, I have completely discussed general problems of which this is a very special case. One of my results is that there is only one number of more than one digit which when multiplied by as many different integers as the number contains digits each product has the same digits as the original number and in the same cyclic order. This number is 142857, which answers the problem.

$$\times 1=142857$$
$$\times 2=285714$$
$$\times 3=428571$$
$$\times 4=571428$$
$$\times 5=714285$$
$$\times 6=857142.$$

Important to note is that the number $\times 7=999{,}999$. If in any of the above six multiples we add the number composed of the first three digits to that composed of the last three, the sum is 999.

II. Solution by the PROPOSER.

Let the digits be a, b, c, d, e, f, and let A, B, C, D, E be some one of the first 6 numbers but 1, respectively; then

$$A(10^5a+10^4b+\ldots\ldots+f)=10^ra-10^{r-1}b+\ldots\ldots+10^{n-r}f$$
$$B(10^5a+10^4b+\ldots\ldots+f)=10^pa-10^{p-1}b+\ \ldots+10^{n-p}f$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$1(10^5a+10^4b+\ldots\ldots+f)=10^5a+10^4b+\ldots\ldots+f$$

$(1+A+B+\ldots\ldots)(10^5a+10^4b+\ldots\ldots+f)=(10^5+10^4+10^3+10^2+10^1+1)$
$(a+b+\ldots\ldots+f)$.

Now $1+A+B+\ldots\ldots=\dfrac{n(n+1)}{2}=21$.

Hence, $10^5a+10^4b+\ldots\ldots+e=\dfrac{111111}{21}(a+b+\ldots\ldots+f)=5291(a+b+\ldots\ldots+f)$.

By subtracting $a+b+\ldots\ldots+e$ from both sides, we get.
$9(9999a+999b+99c+9d+e)=5290(a+b+\ldots\ldots+f)$. Since the left side is divisible by 9, $a+b+c+\ldots\ldots+f$ must be either 27 or 36, but 36 is readily seen to be impossible, since 5291×36 would give a number ending in 6, which when multiplied by six could not give the numbers in rotation; hence the only one to try is 27.

Now $5291\times27=142857$, and this number will be found to answer the purpose.

III. Solution by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

When a common fraction in its lowest terms is changed into a decimal fraction, and this decimal fraction is a pure circulator with a full period, that is, one which begins at the first decimal and contains a number of places by one smaller than the denominator of the common fraction, then the same period will occur, only commencing at a different figure of the period, for every fraction with the same denominator, but a different numerator. The fraction $\frac{1}{7}$ produces a pure circulator with a full period, consequently, according to the principle just mentioned, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ will produce the same period, only commencing at a different figure.

$\frac{1}{7}=.142857$, the next higher figure after 1 in the period is 2,
$\therefore$ $\frac{2}{7}=.285714$, the next higher figure is 4. $\therefore$ $\frac{3}{7}=.428571$, etc. This answers the question proposed, the number being 142857.

IV. Solution by H. C. WHITAKER, B. S., C. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania, and H. W. DRAUGHON, Olio, Mississippi.

It is clear that the first digit is 1, and this can only be the last digit when the multiplier is 3 in which case the last digit of the multiplicand must be 7. Now this 7 is to be multiplied by 2, 4, 5 and 6 and hence the other digits of the required number must be 4, 8, 5 and 2. Now 8 being the largest digit must be the first digit in the product when 6 is the multiplier and hence dividing 8 by 6, we get 4 as the second digit of the required number. Now assume 8 or 5 as the third digit and multiply by 5; the digits can not be brought in the required order; hence the third digit is 2 and the number is 142857.